was progressively reduced. This demonstrated that heat removal by itself is insufficient to size the system. A series of tests was run to determine skin temperatures at which active sweating and shivering begin for various metabolic rates.[5] This testing revealed comfort thresholds illustrated in Fig. 5. The next requirement was to integrate the test results in a concept that could be used to size liquid cooling systems. This was accomplished by designing a system in which the inlet temperature could be varied without changing flow rate. This method of varying LCG inlet temperature is accomplished by a valve which directs all or a portion of the fluid from the LCG to a heat exchanger whose output is mixed with the uncooled fluid and supplied back to the LCG. The selected three-step temperature profile that is within comfort thresholds is shown in Fig. 5 for a fixed flow of 4 lb/min.

Figure 6 illustrates a portion of a manned test conducted at Hamilton Standard utilizing an optimized liquid cooling system. Treadmill speed has varied for a calibrated test subject to determine the ability of the system to maintain crewman comfort when the metabolic rate was step-changed from 1800 to 400 Btu/hr and back to 2200 Btu/hr. Coolant flow was maintained at 4 lb/min and the subject was free to select his cooling garment inlet temperature by manually setting one of three diverter valve positions. Subject comments were noted and liquid temperatures continually monitored throughout the test duration. A point to be noted is the lag of human physiology compared to the response of the mechanical system. Despite these lags, a no-sweat condition was maintained. Figure 5 illustrates the maximum heat removal rate that can be achieved with this liquid cooling system (with a water flow of 4.0 lb/min) while maintaining the subject below the sweating threshold. Although current liquid cooling systems have not demonstrated heat removal rates greatly in excess of 2000 Btu/hr, development of tubing with thinner walls, materials with increased conductivity, and different tubing configurations is expected to result in achievement of heat removal rates in excess of 3000 Btu/hr. Further performance gains are hampered by the body's limitation in transferring heat from the interior to the skin surface. Additionally, as shown in Fig. 2, sustained crewman work rates in excess of 3000 Btu/hr for more than an hour and a half are probably not attainable.

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Attitude Controllability of a Satellite with Flywheels

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FLYWHEEL attitude control systems require the desaturation of angular momentum accumulated by noncyclic external torques. This can be done by control thrust jets or, as recently shown by Frik,[1] using the gravity gradient phenomenon. Usually the control phase and the desaturation phase change periodically. But the satellite control and the flywheel desaturation can be done simultaneously, too. For this purpose, the state vector of the satellite is extended by adding the angular velocity of the flywheels. So both will be controlled, the satellite to the desired position and the wheel velocity to zero, using an external desaturating torque. A convenient way to prove this simultaneous desaturation is with the controllability concept.

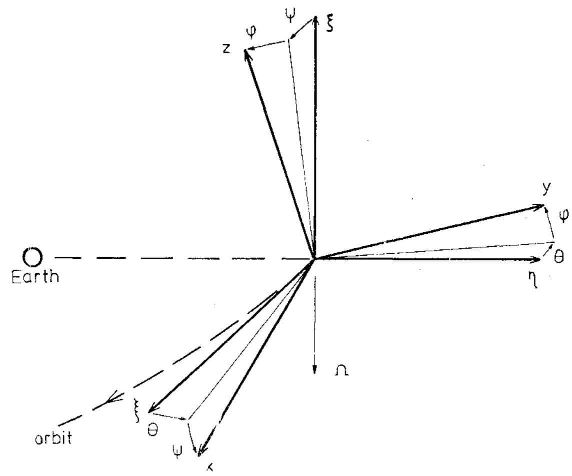

Fig. 1 Reference frame.

Take a satellite containing three flywheels in a circular earth orbit to be positioned in the local vertical, characterized by a reference frame ξ, η, ζ, Fig. 1. The deviations of the satellite from the reference frame are described by pitch angle θ, roll angle ϕ, yaw angle ψ. The flywheel axes are assumed parallel to the satellite principal axes x, y, z. Due to control, angles θ, ϕ, ψ are small all the time.

Euler Equations

$$A_x\dot{\omega}_x + J_x\dot{\nu}_x + (A_z\omega_z + J_z\nu_z)\omega_y - (A_y\omega_y + J_y\nu_y)\omega_z = M_x \tag{1}$$

$(x,y,z$ cyclic perm.)

and the motion equation of the flywheels

$$J_x(\dot{\omega}_x + \dot{\nu}_x) = u_x \qquad (x,y,z \text{ cyclic perm.}) \tag{2}$$

build up our system. A and ω are the satellite principal moments of inertia and angular velocities, respectively, J and ν the same for the flywheels, u the control moment.

It will be proved first that when external torque $M \equiv 0$, the satellite can't be desaturated, i.e., the system isn't controllable. This is the problem of a orbiting vertical-positioned gyrostat without external forces. We introduce the state vector $x = \{\omega_x\omega_y\omega_z'\nu_x\nu_y\nu_z\}^T$, where $\omega_z' = \omega_z + \Omega$ (Ω orbiting velocity), and linearize (1) and (2) about zero equilibrium position.
So we obtain

$$\dot{x} = Ax + Bu \tag{3}$$

$$A = \begin{bmatrix} 0 & a_2 & 0 & 0 & -a_4 & 0 \\ -a_1 & 0 & 0 & a_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -a_2 & 0 & 0 & a_4 & 0 \\ a_1 & 0 & 0 & -a_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} -b_{1x} & 0 & 0 \\ 0 & -b_{1y} & 0 \\ 0 & 0 & -b_{1z} \\ b_{2x} & 0 & 0 \\ 0 & b_{2y} & 0 \\ 0 & 0 & b_{2z} \end{bmatrix}$$

where

$$a_1 = [(A_z - A_x)/(A_y - J_y)]\Omega$$

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$$a_2 = [(A_z - A_y)/(A_x - J_x)]\Omega$$
$$a_3 = [J_x/(A_y - J_y)]\Omega$$
$$a_4 = [J_y/(A_x - J_x)]\Omega$$
$$b_{1x} = [1/(A_x - J_x)],\ b_{2x} = (A_x/J_x)b_{1x}$$

This system is controllable if the matrix $K = [B\ AB \ldots A^5B]$ has the rank equal to the order of the system. Then the nonlinear original system has an open neighborhood of the equilibrium position where it is also controllable.[2]

Regarding (3), it follows, that the rank K is 5: the gyrostat without external forces is not controllable. We get more information from the uncoupled partial systems in (3). Looking for controllability it can be found that the roll-yaw motion $\{\omega_x\omega_y\nu_x\nu_y\}^T$ is controllable, while the pitch motion $\{\omega_z'\nu_z\}^T$ is not. In the roll-yaw case there is a restoring centrifugal torque present causing the controllability of the angular velocities. On the other hand, in the pitch case, no restoring torque is acting: the satellite pitch velocity may be controlled by the flywheel, but the angular momentum corresponding to the z axis never can be changed; that means we can't control ν_z, and it has to be desaturated with an additional restoring external torque.

Secondly, we treat the case $M = M_{\text{Gravity}}$. The linearized expression for M_{Gravity} is

$$M_x = -3\Omega^2(A_z - A_y)\phi,\ M_y = 0,\ M_z = -3\Omega^2(A_x - A_y)\theta \quad (4)$$

There is an external torque acting about the uncoupled pitch motion and therefore full controllability may be supposed for it.

The extended state vector is

$$y = \{\omega_x\omega_y\omega_z'\nu_x\nu_y\nu_z\phi\psi\theta\}^T \quad (5)$$

Contrary to state vector x now the attitude angles ϕ, ψ, θ are considered, too. From (1) and (2), and taking in account the linearized kinematic equations, $\omega_x = \dot{\phi} + \Omega\psi$, $\omega_y = \dot{\psi} - \Omega\phi$, $\omega_z' = \dot{\theta}$, it follows that

$$\dot{y} = A'y + B'u \quad (6)$$

$$A' = \begin{bmatrix} 0 & a_2 & 0 & 0 & -a_4 & 0 & -a_5 & 0 & 0 \\ -a_1 & 0 & 0 & a_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -a_6 \\ 0 & -a_2 & 0 & 0 & a_4 & 0 & a_5 & 0 & 0 \\ a_1 & 0 & 0 & -a_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_6 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & -\Omega & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & \Omega & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where

$$a_5 = 3[(A_z - A_y)/(A_x - J_x)]\Omega^2$$
$$a_6 = 3[(A_x - A_y)/(A_z - J_z)]\Omega^2$$

and the matrix B' is like B with 3 more zero lines.

The controllability matrix $K' = [B'\ A'B' \ldots A'^8B']$ shows us clearly how and where the external torques actuate and influence the rank; we need both M_x and M_z to have a sufficient number of linear independent columns for the rank to be 9. This means that gravity gradient torques make the attitude problem controllable: the desaturation of angular moment is possible for a general three-dimensional motion of the satellite. Further, it can be shown that there is no loss in controllability if only two flywheels are used: one for pitch motion and the other for roll and yaw motion. The gyroscopic coupling between these will take care for the desaturation of angular momentum corresponding to both.

In conclusion, we have treated the controllability of attitude control systems of satellites. Further research has to be done to find a suitable control vector u and the generating control device. A linear control device has been investigated by Schiehlen and Kolbe,[3] which is even applicable to satellites in elliptic orbits, but is not the optimal one.

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Computing Wind Compensated Launcher Settings for Unguided Rockets

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THE first-order approximation of the wind effect on an unguided rocket may be obtained by "wind weighting;" several such procedures[1–4] give approximate launcher adjustments to compensate for the wind effect, but they usually have been restricted to consideration of a specific trajectory objective, or a specific type of trajectory, or both. This Note presents a procedure which allows for a much wider range of applicability. A wind-weighting based model is used as a first approximation, and an iterative procedure is used to refine the numerical values when greater accuracy is required. The use of the complete model as an operational tool requires a real-time computational capability, c.f. Duncan and Rachele.[5]

Coordinate Systems and Transformations

The rocket trajectory is specified in a right-hand topocentric coordinate system (X,Y,Z). The positive X-axis points east and the positive Y-axis points north. The azimuth angle, α, and the elevation angle, θ, are defined in Fig. 1; θ_1 and θ_2, are the components of θ in the YZ and XZ planes, respectively.

Table 1 Change in burnout attitude and impact for specific changes in launch angles (Regular Athena)

Burnout				Impact	
$\Delta\theta_1$, rad	$\Delta\theta_2$, rad	$\Delta\theta_{1b}$, μrad	$\Delta\theta_{2b}$, μrad	ΔX, km	ΔY, km
0.02	0.00	57063	6082	14.48	34.85
0.01	0.00	28378	3154	7.32	16.63
−0.01	0.00	−28068	3392	−7.46	−14.82
−0.02	0.00	−55821	7035	−15.07	−27.98
0.00	0.02	−5573	65389	51.36	13.94
0.00	0.01	−2662	32854	26.14	6.69
0.00	−0.01	2420	−33154	−27.00	−5.93
0.00	−0.02	4602	−66587	−54.77	−11.28
0.01	0.01	25773	29452	34.04	23.06
−0.01	0.01	−30780	36510	18.08	−8.01
0.01	−0.01	30747	−36046	−20.28	10.82
−0.01	−0.01	−25603	−30040	−33.84	−20.98

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